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Reconciling neutralino relic density with Yukawa unified supersymmetric models

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Abstract: Supersymmetric grand unified models based on the gauge group SO(10) are especially attractive in light of recent data on neutrino masses. The simplest SO(10) SUSY GUT models predict unification of third generation Yukawa couplings in addition to the usual gauge coupling unification. Recent surveys of Yukawa unified SUSY GUT models predict an inverted scalar mass hierarchy in the spectrum of sparticle masses if the superpotential μ term is positive. In general, such models tend to predict an overabundance of dark matter in the universe. We survey several solutions to the dark matter problem in Yukawa unified supersymmetric models. One solution — lowering the GUT scale mass value of first and second generation scalars — leads to $\tilde{u}_R$ and $\tilde{c}_R$ squark masses in the $90-120$ GeV regime, which should be accessible to Fermilab Tevatron experiments. We also examine relaxing gaugino mass universality which may solve the relic density problem by having neutralino annihilations via the Z or h resonances, or by having a wino-like LSP.

Keywords: Supersymmetric Standard Model, GUT, Hadronic Colliders.

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Contents

1. Introduction

The unification of gauge couplings in the Minimal Supersymmetric Standard Model (MSSM) can be construed as indirect evidence for the existence of weak scale supersymmetry. It is of course also suggestive of the existence of a grand unified symmetry at energy scales $Q \geq M_{\rm GUT} \simeq 2 \times 10^{16}$ GeV. Such a SUSY GUT theory may be the "low energy effective theory" that can be obtained from some more fundamental superstring theory. Models based on the gauge group SU(5) are compelling in that they explain the apparently ad-hoc hypercharge assignments of the SM fermions [1]. However, many SU(5) SUSY GUT models as formulated in four dimensions are already excluded by proton decay constraints [2]. SU(5) SUSY GUT models can also be formulated in five or more dimensions, where compactification of the extra dimensions leads to a break down in gauge symmetry [3]. These models can dispense with the unwieldy large Higgs representations required by four dimensional models, and can also be constructed to suppress or eliminate proton decay entirely.

The steady accumulation of data on neutrino mass and neutrino oscillations in the past several years has given strong motivation to the possibility that nature is described by a SUSY GUT theory based on the gauge group SO(10) [4, 5]. In SO(10) theories, all 15 of the SM matter fields of a single generation inhabit the 16-dimensional spinorial representation of SO(10). The 16th element can be taken to be a SM gauge singlet field that functions as the right-hand neutrino. A large Majorana mass term is allowed for this field by the gauge symmetry. Dominantly left handed neutrinos thus gain tiny masses via the see-saw mechanism, while dominantly right handed neutrinos acquire a mass near to the unification scale, and are conveniently swept out-of-sight [6]. The neutrino see-saw mechanism also lends itself elegantly to a mechanism for baryogenesis via leptogenesis [7]. For these reasons (and others), the possibility that nature is described by an SO(10) SUSY GUT model at scales $Q \geq M_{\rm GUT}$ deserves serious consideration.

In the simplest SO(10) SUSY GUT models, the two Higgs doublets of the MSSM occupy the same 10 dimensional Higgs multiplet $\phi(\mathbf{10})$. The superpotential then contains the term

$$\hat{f} \ni f\psi(\mathbf{16})\psi(\mathbf{16})\phi(\mathbf{10}) + \cdots \tag{1.1}$$

where f is the single Yukawa coupling for the third generation. Thus, the simplest SO(10) SUSY GUT models predict $t-b-\tau$ Yukawa coupling unification in addition to gauge coupling unification. It is possible to calculate the t, b and τ Yukawa couplings at $Q = m_{weak}$, and extrapolate them to $M_{\rm GUT}$ in much the same way one checks the theory for gauge coupling unification. Yukawa coupling unification turns out to depend on the entire spectrum of sparticle masses and mixings [8] since these enter into the weak scale supersymmetric threshold corrections. Thus, the requirement of $t-b-\tau$ Yukawa coupling unification can be a powerful constraint on the soft SUSY breaking terms of the low energy effective theory [9].

It is well known from early work that large values of $\tan\beta \sim 50$ are required for $t-b-\tau$ unification. However, in models with a common scalar mass, such as mSUGRA, it was found that a breakdown in the mechanism of radiative EWSB occurred which excluded $\tan\beta$ values which would give rise to Yukawa coupling unification. In ref. [10], it was found that models with Yukawa coupling unification with $R < 1.05$ (good to 5%) could be found if additional D-term splittings of scalar masses were included. Here, $R \equiv max(f_t,\ f_b,\ f_\tau)/min(f_t,\ f_b,\ f_\tau)$, where all Yukawa couplings are evaluated at the GUT scale. The D-term splittings occur naturally when the SO(10) gauge symmetry breaks to SU(5), and they are given by [11]

$$\begin{aligned} m_Q^2 = m_E^2 = m_U^2 &= m_{16}^2 + M_D^2\,, \\ m_D^2 = m_L^2 &= m_{16}^2 - 3M_D^2\,, \\ m_N^2 &= m_{16}^2 + 5M_D^2\,, \\ m_{H_{u,d}}^2 &= m_{10}^2 \mp 2M_D^2\,, \end{aligned} \tag{1.2}$$

where M_D^2 parameterizes the magnitude of the D-terms. Owing to our ignorance of the gauge symmetry breaking mechanism, M_D^2 can be taken as a free parameter, with either positive or negative values. $|M_D|$ is expected to be of order the weak scale. Thus, the D-term (DT) model is characterized by the following free parameters,

$$m_{16}\,,\ m_{10}\,,\ M_D^2\,,\ m_{1/2}\,,\ A_0\,,\ \tan\beta\,,\ \mathrm{sign}(\mu)\,. \tag{1.3}$$

Using the DT model, Yukawa unification good to 5% was found when soft term parameters m_{16} and m_{10} were scanned up to 1.5 TeV, but only for $\mu < 0$ values [10]. The essential quality of the D-term mass splitting is that it gave the value of m_{H_u} a head start over m_{H_d} in running towards negative values, as is required for REWSB. In ref. [12], the DT model for $\mu < 0$ was explored in more detail, and the neutralino relic density was explored. It was found that a good relic density could be found in the A-annihilation funnel [13], although a rather large value of $BF(b \to s\gamma) \gtrsim 4 \times 10^{-4}$ was usually predicted. Good relic density was also found in the stau co-annihilation [14] and hyperbolic branch/focus point (HB/FP) region [15, 16, 17], but at some cost to the degree of Yukawa coupling unification.[1]

Later, E821 measurements of the muon anomalous magnetic moment appeared, which favored positive values of the superpotential μ parameter. In ref. [18], it was found that Yukawa coupling unification good to only 30% could be achieved in DT models with $\mu > 0$ when m_{16} values were scanned up to 2 TeV. The models with the best Yukawa coupling unification were found to have soft term relations

$$A_0^2 \simeq 2m_{10}^2 \simeq 4m_{16}^2\,, \tag{1.4}$$

which had also been found by Bagger *et al.* in the context of radiatively driven inverted scalar mass hierarchy (IMH) models [19, 20, 21].[2] In ref. [18], a model with just GUT scale Higgs mass splittings was also examined (HS), while all other scalars remained universal. The parameter space of the HS model is that of eq. 1.3, but where the D-term splitting is only applied to the last of the relations in eq. 1.2. Yukawa coupling unification in the HS model was found to be comparable to the DT model case when m_{16} values up to 2 TeV were scanned. Finally, in ref. [22] (Auto *et al.*), soft term values of m_{16} up to 20 TeV were explored. In this case, Yukawa unified solutions to better than 5% were found for $\mu > 0$ for the HS model when very large values of $m_{16} > 5-10$ TeV were scanned. The large scalar masses that gave rise to Yukawa unification also acted to suppress neutralino annihilation in the early universe, so that rather large values of the relic density were found. Models with $\Omega_{\widetilde{Z}_1}h^2 < 0.2$ could be found, but only at the expense of accepting Yukawa coupling unification to 20%, rather than 5%. The models found with low relic density generally had either a low μ value, or were in the light Higgs annihilation corridor, with $2m_{\widetilde{Z}_1} \sim m_h$.

Similar work has been carried on by Blazek, Dermisek and Raby (BDR). In ref. [23, 24], the BDR group used a top-down approach to the RG sparticle mass solution to find Yukawa unified solutions for $\mu > 0$, where they also noted that in this case the HS model worked better than the DT model. In their approach, the third generation fermion masses and other electroweak observables were an output of the program, so that starting with models with perfect Yukawa coupling unification, they would look for solutions with a low χ^2 value constructed from the low energy observables. The BDR Yukawa unified solutions were also characterized by soft term IMH model boundary conditions. The solutions differed from those of ref. [22] in that they always gave a very low value of Higgs mass m_A and also small μ parameter, indicative of a mixed higgsino-bino LSP. In ref. [25], the neutralino relic density was examined for the BDR solutions. Their low μ and m_A values generally led to very low values of $\Omega_{\widetilde{Z}_1}h^2$ unless $m_{1/2}$ was small enough compared to μ that the LSP was in the mixed higgsino-bino region.[3]

[1]The HB/FP region is characterized by $|\mu| \lesssim M_2$ so that the LSP is a mixed higgsino-bino, and has a large annihilation rate into vector bosons.

[2]In IMH models, starting with multi-TeV scalars masses at $Q = M_{\rm GUT}$, for certain choices of boundary conditions third generation and Higgs soft masses are driven to sub-TeV values, while first and second generation scalars remain heavy.

[3]We note that the HS model *without* Yukawa coupling unification has recently been examined by Ellis *et al.*, ref. [26].

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In this paper, we give further scrutiny to the Auto *et al.* Yukawa unified solutions, especially with respect to the relic density of neutralinos expected from these models. Our goal is to find regions of parameter space where a high degree of Yukawa coupling unification is preserved, but also where a cold dark matter relic density in accord with WMAP measurements [27] can be found.[4] To do so, we expand the parameter space of the SO(10) model. In section 2, we consider relaxing generational universality of the matter scalars. The expansion of parameter space in this fashion is consistent with the SO(10) gauge symmetry, which only preserves universality within a generation (when SO(10) is unbroken). By maintaining TeV scale third generation masses, but reducing first and second generation masses, light first and second generation scalars are produced which enhance the neutralino annihilation rate. The curious effect in the HS model is that the split Higgs mass contribution to RG running causes the right up and charm squarks to be by far the lightest of the scalars, with $m_{\tilde{u}_R}$ and $m_{\tilde{c}_R} \sim 100\,\mathrm{GeV}$ needed to achieve the required relic density. We show that such light squarks are just at the limit of observability for the Fermilab Tevatron collider. Our analysis motivates an experimental search for just two species of light squarks, while all other scalars are in the TeV range, and where $m_{\tilde{g}} \sim 400\,\mathrm{GeV}$.

In section 3, instead we relax the condition of gaugino mass universality. The gauge kinetic function of supergravity theories need not adopt its minimal form in order to respect the SO(10) gauge symmetry. We examine several models found in ref. [22], and vary the U(1) gaugino mass M_1. This allows the bino-like LSP to sit atop the h or Z resonance, which in some cases is enough to reduce the relic density to the $\Omega_{\widetilde{Z}_1} h^2 \sim 0.1$ level. Also, by increasing substantially the value of M_1, ultimately the lightest neutralino becomes wino-like, and a valid relic density is also achieved. Variation of the gaugino masses preserves the Yukawa coupling unification in all the models we examined. We also examine the case of lowering the SU(2) gaugino mass M_2 to obtain a wino-like LSP. In this case, the lightest chargino usually drops below limits from LEP2. In section 4, we present a summary and conclusions. We note here additional studies which have been performed, where just $b-\tau$ unification was examined [28, 29] or where Yukawa quasi-unification was examined [30].

2. Scalar mass non-universality and light squarks

Our goal in this section is to explore whether an expanded parameter space allowing non-universality of generations can provide new avenues towards solving the dark matter problem in Yukawa unified supersymmetric models. Thus, we expand the parameter space of the HS model to now include

$$m_{16}(1)\,,\ m_{16}(3)\,,\ m_{10}\,,\ M_D^2\,,\ m_{1/2}\,,\ A_0\,,\ \tan\beta \quad \text{and} \quad sign(\mu), \tag{2.1}$$

where the matter scalar soft mass is now broken up into $m_{16}(3)$ for the third generation, and $m_{16}(1)$ for the first generation. We will also assume $m_{16}(2) = m_{16}(1)$ to reduce parameter space freedom and to quell potential contributions to flavor changing processes

[4]The WMAP collaboration has determined $\Omega_{CDM} h^2 = 0.1126^{+0.0161}_{-0.0181}$ at 2σ level.

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parameter	value (GeV)
$m_{16}(3)$	7826.5
$m_{16}(1)$	1202.0
m_{10}	9652.3
M_D^2	$(2894.9)^2$
$m_{1/2}$	80.0
A_0	-16626.0
$\tan\beta$	51.165
$f_t(M_{\rm GUT})$	0.563
$f_b(M_{\rm GUT})$	0.549
$f_\tau(M_{\rm GUT})$	0.563
R	1.02
μ	3075.4
$m_{\tilde g}$	380.1
$m_{\tilde u_L}$	1172.2
$m_{\tilde u_R}$	107.0
$m_{\tilde d_L}$	1175.0

parameter	value (GeV)
$m_{\tilde d_R}$	1343.7
$m_{\tilde e_L}$	693.4
$m_{\tilde e_R}$	1734.2
$m_{\tilde\nu_L}$	672.7
$m_{\tilde t_1}$	1651.1
$m_{\tilde b_1}$	2330.5
$m_{\tilde\tau_1}$	3049.2
$m_{\widetilde W_1}$	135.1
$m_{\widetilde Z_1}$	57.3
$m_{\widetilde Z_2}$	134.9
m_A	2505.7
m_h	134.4
$\Omega_{\widetilde Z_1}h^2$	0.093
$BF(b\to s\gamma)$	3.96×10^{-4}
Δa_μ	15.7×10^{-10}

Table 1: Masses and parameters in GeV units for a Yukawa unified HS model with generational non-universality. The spectrum is obtained using ISAJET v7.69.

like $K_L - K_S$ mass difference and $BF(\mu\to e\gamma)$. Allowing third generation non-universality can also contribute to the $\Delta m_B \equiv m^0_{B_H} - m^0_{B_L}$ mass difference, the rate $BF(\tau\to e\gamma)$ or $BF(\tau\to\mu\gamma)$, and $BF(b\to s\gamma)$. However, these latter constraints are all rather mild, and in general can allow for significant deviations in generational non-universality [31, 32, 33].

Our goal is to maintain the successful prediction of Yukawa coupling unification found in ref. [22] with heavy scalars and an inverted scalar mass hierarchy, but at the same time reduce some of the scalar particle masses so that there is not too severe a suppression of neutralino annihilation in the early universe which leads to too large a relic density. The IMH mechanism of Bagger *et al.* works if the mass relation between third generation scalars, Higgses and A terms is preserved: $A_0^2 \simeq 2m_{10}^2 \simeq 4m_{16}^2(3)$. Thus, a possible way forward is to decrease the value $m_{16}(1) = m_{16}(2)$ so far that a valid relic density is attained. It turns out this procedure is possible, but that it leads to a qualitatively new spectrum of superparticle masses wherein two of the squarks — $\tilde u_R$ and $\tilde c_R$ — become quite light, in the 90-120 GeV regime, while the other scalar masses remain quite heavy.

A sample spectrum is shown in table 1, which uses Isajet 7.69 to generate the spectrum [34]. As can be seen, a high degree of Yukawa coupling unification is maintained, while simultaneously achieving an acceptable dark matter relic density $\Omega_{\widetilde Z_1}h^2 = 0.09$. The low relic density is obtained because the light $\tilde u_R$ and $\tilde c_R$ squarks allow $\widetilde Z_1\widetilde Z_1 \to u\bar u$, $c\bar c$ annihilations to proceed at a high rate due to unsuppressed t-channel squark exchange.

Why are some of the squark masses quite light, while most other scalars are heavy? The answer comes from the 1-loop RGEs for the SUSY soft breaking terms. For first generation matter scalars, they have the form [35]

$$\frac{dm_{Q_1}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{1}{15}g_1^2M_1^2 - 3g_2^2M_2^2 - \frac{16}{3}g_3^2M_3^2 + \frac{1}{10}g_1^2S\right), \tag{2.2}$$

$$\frac{dm_{U_1}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{16}{15}g_1^2M_1^2 - \frac{16}{3}g_3^2M_3^2 - \frac{2}{5}g_1^2S\right), \tag{2.3}$$

$$\frac{dm_{D_1}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{4}{15}g_1^2M_1^2 - \frac{16}{3}g_3^2M_3^2 + \frac{1}{5}g_1^2S\right), \tag{2.4}$$

$$\frac{dm_{L_1}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{3}{5}g_1^2M_1^2 - 3g_2^2M_2^2 - \frac{3}{10}g_1^2S\right), \tag{2.5}$$

$$\frac{dm_{E_1}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{12}{5}g_1^2M_1^2 + \frac{3}{5}g_1^2S\right), \tag{2.6}$$

where

$$S = m_{H_u}^2 - m_{H_d}^2 + Tr\left[\mathbf{m}_Q^2 - \mathbf{m}_L^2 - 2\mathbf{m}_U^2 + \mathbf{m}_D^2 + \mathbf{m}_E^2\right]. \tag{2.7}$$

In the mSUGRA model, the term involving S doesn't contribute to RG running at all, since universality implies $S = 0$. However, in the HS model, while the trace over mass matrices remains zero, the contribution from Higgs splitting is large, especially for our case where the soft Higgs masses can be $\sim 10\,\text{TeV}$. Radiative EWSB in Yukawa unified models requires $m_{H_d}^2 > m_{H_u}^2$, so that in fact $S < 0$. Then the soft masses with negative co-efficients on the S terms will be highly suppressed in their RG running, while terms with positive S coefficient will be pushed to higher mass values. If the S term dominates the RG running, then indeed we expect a large splitting due to RG evolution in all the first and second generation soft breaking masses, with the splitting being dictated by the soft mass $\mathrm{U}(1)_Y$ quantum numbers. The splitting amongst third generation soft masses will be much less, since their RGEs include additional Yukawa coupling times soft mass terms, and the third generation soft terms are also large.

In figure 1a., we show the third generation and Higgs soft term running, for the same case as in table 1. The soft term suppression due to Yukawa coupling contributions is apparent. In figure 1b., we show first generation soft term running. In this case, the large splitting due to the dominant S term is evident, and causes the U_1 mass to run to very low values, which gives rise to the light $\tilde{u}_R$ and $\tilde{c}_R$ squarks. In addition, the L_1 soft mass also is suppressed, though not as much as U_1, so that rather light left selectrons and smuons and their associated sneutrinos are generated.

In figure 2, we show the $m_{16}(1)$ *vs.* $m_{1/2}$ plane for fixed $m_{16}(3) = 7826.5\,\text{GeV}$, and other parameters as given in table 1. The blue region at low $m_{1/2}$ is excluded by the LEP2 bound on chargino mass $m_{\widetilde{W}_1} > 103.5\,\text{GeV}$. The black region on the left is excluded because the $m_{U_1}^2$ squared mass is driven tachyonic, resulting in a color symmetry violating ground state. The green region denotes sparticle mass spectrum solution with relic density $0.094 < \Omega_{\widetilde{Z}_1}h^2 < 0.129$, within the WMAP favored regime, while the yellow region denotes even lower relic density values, wherein the CDM in the universe might be a mixture of neutralinos plus some other species. The red region has $\Omega_{\widetilde{Z}_1}h^2 < 0.5$. It may be allowed if

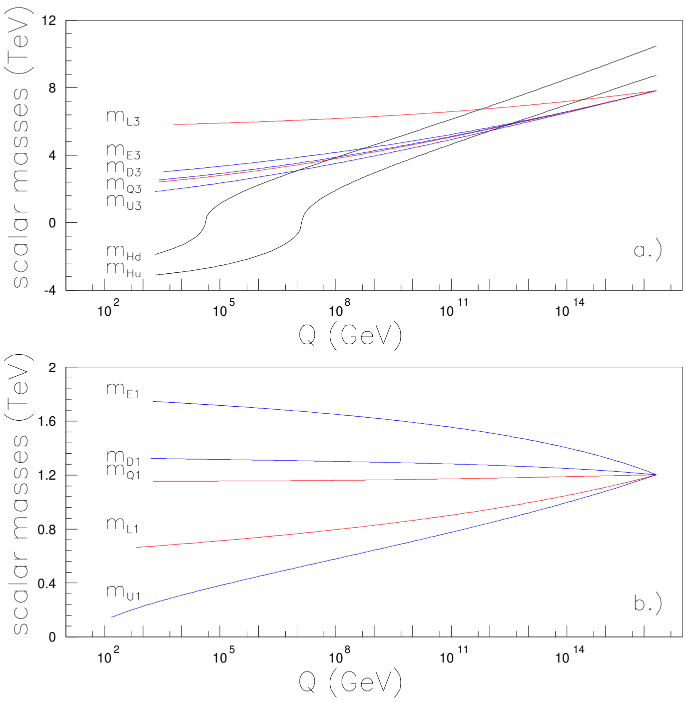

Figure 1: Evolution of (a) third generation and Higgs scalar soft masses and (b) first generation soft masses for the case of $m_{16}(3) = 7826.5\,\mathrm{GeV}$, $m_{10} = 9652.3\,\mathrm{GeV}$, $M_D = 2894.9\,\mathrm{GeV}$, $m_{16}(1) = 1202.3\,\mathrm{GeV}$, $m_{1/2} = 80\,\mathrm{GeV}$, $A_0 = -16626\,\mathrm{GeV}$, $\mu > 0$, $\tan\beta = 51.165$ and $m_t = 180\,\mathrm{GeV}$. For Higgs soft masses, we actually plot $sign(m_H^2) \cdot \sqrt{|m_H^2|}$.

QCD corrections to $\widetilde{Z}_1\widetilde{Z}_1 \to u\bar{u}$, $c\bar{c}$ are large and positive.[5] The remaining unshaded regions give sparticle spectra solutions with $\Omega_{\widetilde{Z}_1}h^2 > 0.5$, and would be excluded by WMAP. We also show several contours of $BF(b \to s\gamma)$, Δa_μ^{SUSY} and R, the max/min ratio of GUT scale Yukawa couplings. We see that all constraints on the sparticle mass spectrum are within allowable limits for the shaded region to the right of the excluded region.

We have also performed a χ^2 analysis in the parameter space listed in eq. 2.1 and scanned over the following range of seven parameters:

$$\begin{aligned} &0 < m_{16}(1) < 10\,TeV\,, \quad 0 < m_{16}(3) < 10\,TeV\,, \quad 0 < m_{10} < 15\,TeV\,, \\ &0 < m_{1/2} < 0.5\,TeV\,, \quad 0 < M_D < 5\,TeV\,, \quad -30 < A_0 < 30\,TeV\,, \quad 40 < \tan\beta < 60 \end{aligned} \tag{2.8}$$

For every point in parameter space, we calculate the χ^2 quantity formed from Δa_μ, $BF(b \to s\gamma)$ and $\Omega_{\widetilde{Z}_1}h^2$. We present plots with a color coded representation of the χ^2 value, where red shading corresponds to high χ^2, green shading corresponds to low χ^2 and yellow shading has intermediate values. The numerical correspondence is listed in figure 3. As a result, we have found that in order to have Yukawa unification below $\sim 5\%$ level and low χ^2, one should be in the special, strongly correlated region of the parameter space. For further

[5] A subset of QCD corrections for $\widetilde{Z}_1\widetilde{Z}_1 \to q\bar{q}$ has been calculated by Drees *et al.*, and are found to be small [36].

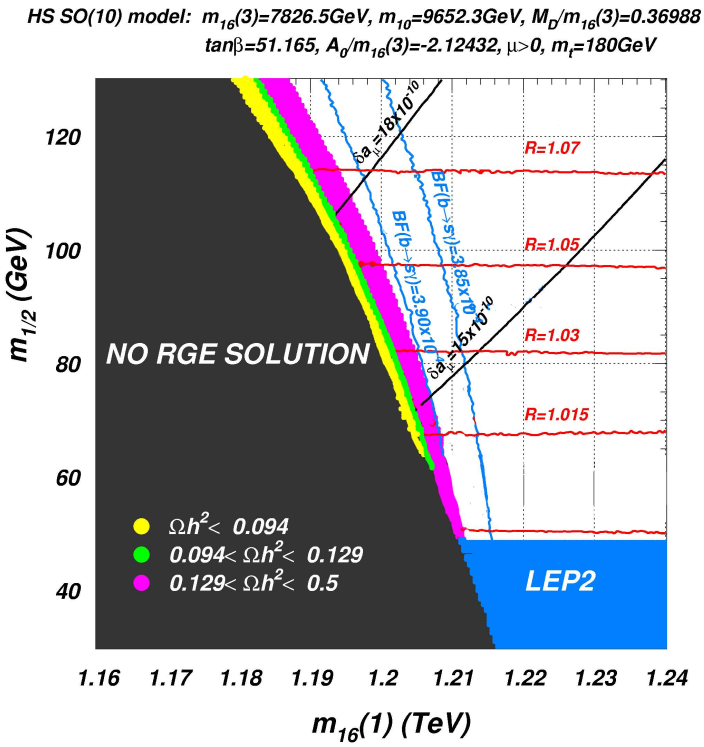

Figure 2: Allowed parameter space of Yukawa unified supersymmetric HS model with generational non-universality. We show the $m_{16}(1)$ *vs.* $m_{1/2}$ plane for $m_{16}(3) = 7830\,\text{GeV}$, $m_{10} = 9650\,\text{GeV}$, $M_D/m_{16}(3) = 0.37$, $A_0/m_{16}(3) = -2.1$, $\mu > 0$, $\tan\beta = 51$ and $m_t = 180\,\text{GeV}$. The black shaded region gives tachyonic particles, while the blue region is excluded by LEP2 chargino search experiments. The yellow and green regions are allowed by the WMAP determination of $\Omega_{\widetilde{Z}_1}h^2$. We also show contours of R, the measure of Yukawa unification at M_{GUT}.

analysis we have fixed some parameters at the values favored by the χ^2 analysis:

$$m_{16}(3) = 7750\,\text{GeV}\,, \qquad A_0 = -2.05 m_{16}(3)\,, \tag{2.9}$$

and varied the others within the χ^2 fit preferred range:

$$\begin{aligned} &49.8 \leq \tan\beta \leq 51.6\,, && 75\,\text{GeV} \leq m_{1/2} \leq 120\,\text{GeV}\,, \\ &0.14 \leq \frac{m_{H_d} - m_{H_u}}{m_{16}(3)} \leq 0.23\,, && 1.225 \leq \frac{m_{H_d} + m_{H_u}}{2m_{16}(3)} \leq 1.27\,. \end{aligned} \tag{2.10}$$

Finally, for each point generated in the range specified by eq. 2.10 the value of χ^2 was minimized by varying $m_{16}(1)$.

Those solutions and respective correlations among the model parameters are illustrated in figure 3 where we show the results of scanning over the complete parameter space listed in eq. 2.8. The color coding of the points corresponds to a χ^2 value computed from the value of $\Omega_{\widetilde{Z}_1}h^2$, Δa_μ^{SUSY} and $BF(b \to s\gamma)$. In constructing the χ^2 quantity, we adopt the limits that $\Omega_{\widetilde{Z}_1}h^2 = 0.1126 \pm 0.00805$ (we use the upper limit only), $\Delta a_\mu^{SUSY} = (27 \pm 10) \times 10^{-10}$ (as required by the recent E821 $(g-2)_\mu$ measurement compared to SM predictions [37]), and $BF(b \to s\gamma) = (3.25 \pm 0.54) \times 10^{-4}$ [38].

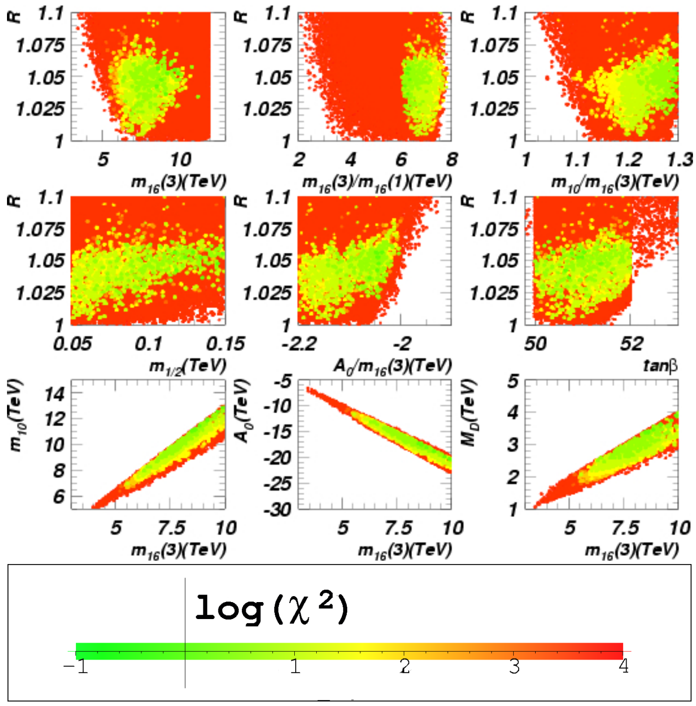

Figure 3: Scan of SO(10) HS models with generational non-universality for various planes of the parameter space. The colors of the dots correspond to the χ^2 value computed from $\Omega_{\widetilde{Z}_1}h^2$, $BF(b\to s\gamma)$ and a_μ^{SUSY}.

In figure 4, we show the results of scanning over the narrowed parameter space defined by eq. 2.10 in the R *vs.* $\Omega_{\widetilde{Z}_1}h^2$ plane with the same color coding as in figure 3. We see that a large range of points are generated with low χ^2 value, and that many of these have Yukawa coupling unification below the 5% level, and also with a WMAP allowed relic density.

In figure 5, we show the same points from the parameter space scan, but this time in the $m_{\tilde{u}_R}$ *vs.* R plane. In this case, we see that essentially all of the low χ^2 points have $m_{\tilde{u}_R}$ values in the 90-120 GeV range. For models with $m_{\tilde{u}_R} > 120$ GeV, the value of $\Omega_{\widetilde{Z}_1}h^2$ exceeds the WMAP bounds, and the models become excluded. The allowed models shown all have R values in the 1.02-1.21 range.

2.1 Phenomenological consequences of two light squarks

Given that the Yukawa unified model with split generations predicts two light squarks of mass 90-120 GeV in order to obtain a relic density in accord with WMAP, it is natural to examine the phenomenological consequences of such light squarks. We first turn to data from LEP2. In ref. [39], the ALEPH collaboration reports mass limits on squarks in the MSSM. Using 410 pb^{-1} of data ranging up to CM energies of 201.6 GeV, they report $m_{\tilde{q}} > 97$ GeV at 95% CL assuming $\tilde{q}\to q\widetilde{Z}_1$ with a mass gap $m_{\tilde{q}} - m_{\widetilde{Z}_1} > 6$ GeV, and assuming 10 species of degenerate squarks. In our case, with just two nearly degenerate squarks, the mass limits will be even lower.[6] Furthermore, the OPAL collaboration [40] has

[6] If we make the simplifying assumption that all of the mass limit comes from running at the highest

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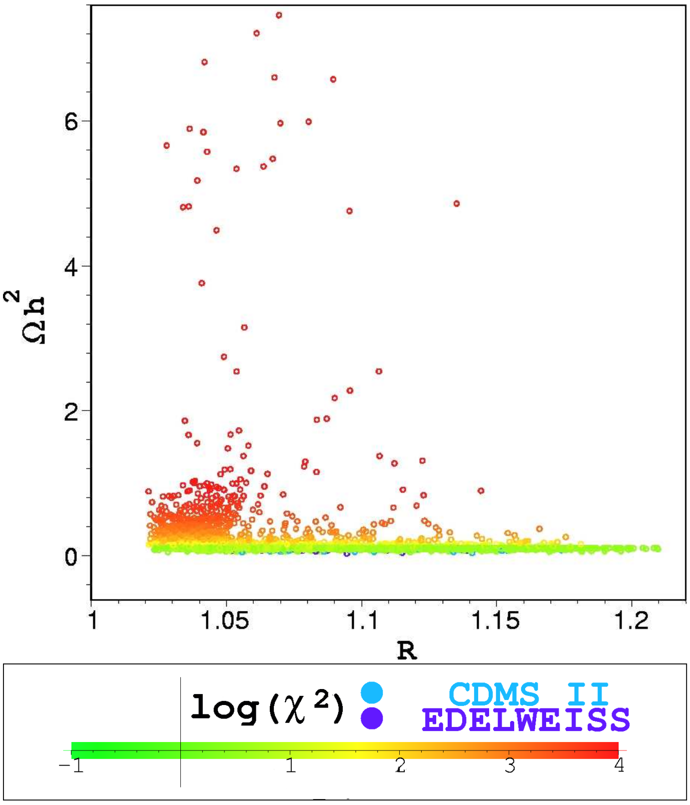

Figure 4: Scan of SO(10) HS models with generational non-universality, in the R *vs.* $\Omega_{\widetilde{Z}_1}h^2$ plane. The colors of the dots correspond to the χ^2 value computed from $\Omega_{\widetilde{Z}_1}h^2$, $BF(b \to s\gamma)$ and a_μ^{SUSY}. Points excluded by Edelweiss (CDMS II) direct dark matter searches are denoted by purple (blue) dots.

searched for $\tilde{t}_1\bar{\tilde{t}}_1$ pairs assuming $\tilde{t}_1 \to c\widetilde{Z}_1$. These limits exclude can be applied to our case of $\tilde{c}_R\bar{\tilde{c}}_R$ production, and exclude $m_{\tilde{c}_R} < 90\,\text{GeV}$ for almost all values of $m_{\widetilde{Z}_1} < m_{\tilde{c}_R} - m_c$.

There are also limits on squark pair production from Fermilab Tevatron experiments. In ref. [41], the CDF collaboration reports a mass limit of $m_{\tilde{q}} \gtrsim 250\,\text{GeV}$ when $m_{\tilde{g}} \simeq 400\,\text{GeV}$, based on 84 pb^{-1} of data taken in Run 1 with $\sqrt{s} = 1.8\,\text{TeV}$. This analysis assumes eight degenerate squarks when MSSM parameters are used, and can potentially rule out the case presented here with two species of nearly degenerate squarks. Thus, we examine the relative signal produced in the Yukawa unified case with two light squarks with mass $\sim 90 - 120\,\text{GeV}$. A similar analysis by the D0 collaboration [42] also excludes $m_{\tilde{q}} < 250\,\text{GeV}$ (with 10 degenerate squarks) by searching for jets+ $\not{E}_T$ events, using 79 pb^{-1} of Run 1 data. This analysis assumes the mSUGRA model, however, which doesn't give rise to our case where $m_{\tilde{q}} \ll m_{\tilde{g}}$.

We generate all sparticle pair production reactions using the event generator Isajet 7.69, using the model parameters as given in table 1, except allowing $m_{16}(1)$ to float as a free parameter. The mass value $m_{\tilde{u}_R}$ varies nearly linearly with $m_{16}(1)$, so this allows us to vary

energy of $\sqrt{s} = 201.6\,\text{GeV}$, then the 97 GeV mass limit for 10 degenerate squarks would be reduced to 89 GeV if only $\tilde{u}_R\bar{\tilde{u}}_R$ and $\tilde{c}_R\bar{\tilde{c}}_R$ are produced.

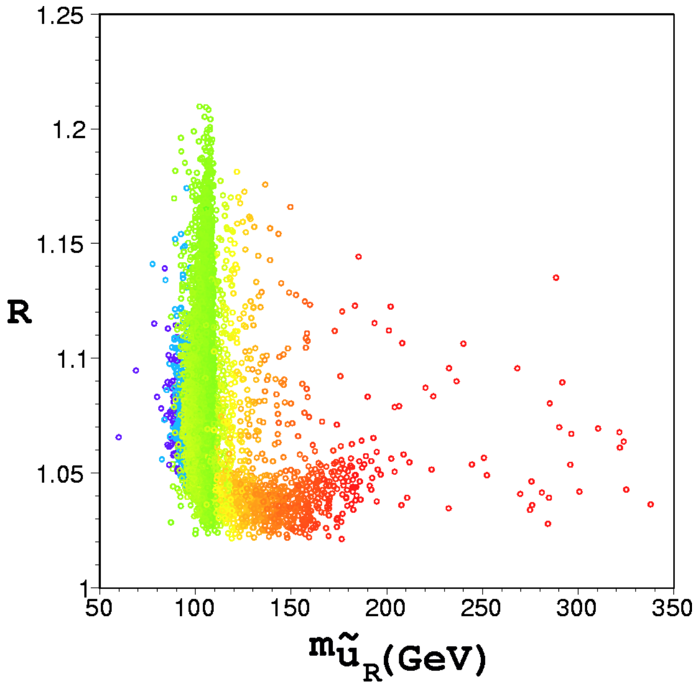

Figure 5: Scan of SO(10) HS models with generational non-universality, in the R *vs.* $m_{\tilde{u}_R}$ plane. The colors of the dots correspond to the χ^2 value as listed in figure 4.

the squark masses, while keeping other parameters fixed. We use the ISAJET toy detector CALSIM with calorimetry covering the regions $-4 < \eta < 4$ with cell size $\Delta\eta \times \Delta\phi = 0.1 \times 0.262$. Electromagnetic energy resolution is given by $\Delta E_{em}/E_{em} = 0.15/\sqrt{E_{em}}$, while hadronic resolution is given by $\Delta E_h/E_h = 0.7/\sqrt{E_h}$. Jets are identified using the ISAJET jet finding algorithm GETJET using a fixed cone size of $\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2} = 0.7$. Clusters with $E_T > 15\,\mathrm{GeV}$ and $|\eta(jet)| < 2.5$ are labeled as jets. Muons and electrons are classified as isolated if they have $E_T > 5\,\mathrm{GeV}$, $|\eta_\ell| < 2.5$, and the visible activity within a cone of $R = 0.4$ about the lepton direction is less than $2\,\mathrm{GeV}$.

The CDF analysis requires the following cuts:

- $N(\text{jets}) \geq 3$,
- at least one jet has $|\eta| < 1.1$,
- no isolated leptons,
- the second and third highest $\not{E}_T$ jets are fiducial,
- $R_1 \equiv \sqrt{\delta\phi_2^2 + (\pi - \delta\phi_1^2)} > 0.75$ rad, $R_2 \equiv \sqrt{\delta\phi_1^2 + (\pi - \delta\phi_2^2)} > 0.5$ rad ($\delta\phi_1 = |\phi_{\text{leading jet}} - \phi_{\not{E}_T}|$ and $\delta\phi_2 = |\phi_{\text{second jet}} - \phi_{\not{E}_T}|$),
- $\not{E}_T > 70\,\mathrm{GeV}$,
- $H_T \equiv \not{E}_T + E_T(jet\ 2) + E_T(jet\ 3) \geq 150\,\mathrm{GeV}$,

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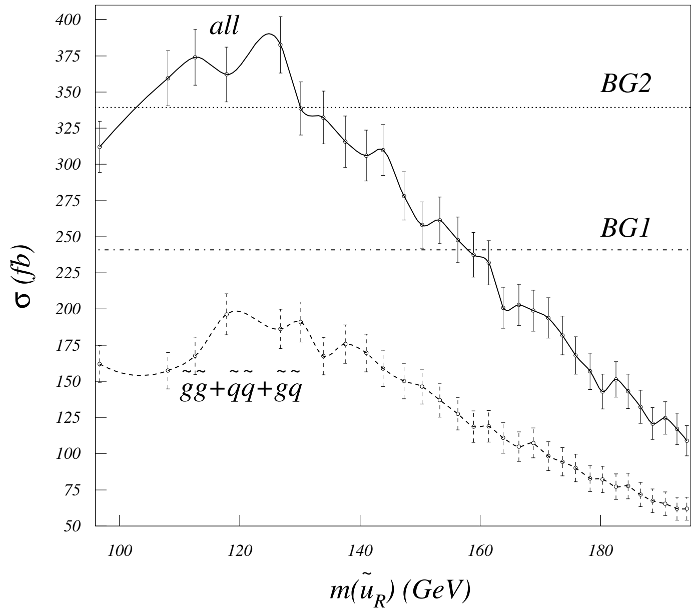

Figure 6: Plot of cross section for ≥ 3 jet+ $\not{E}_T$ events *vs.* $m_{\tilde{u}_R}$ at the Fermilab Tevatron collider after implementing CDF cuts, for same case as in table 1, but with varying $m_{16}(1)$. We show plots assuming all sparticles contributing to the cross section, and also the cross section coming just from gluino and squark production. We also show the 95% CL signal level for errors added linearly (upper) or in quadrature (lower).

which we also implement into our analysis. In figure 6, we plot the signal cross section after cuts. The dashed curve is the result including just $\tilde{q}\tilde{q}+\tilde{g}\tilde{q}+\tilde{g}\tilde{g}$ production (as CDF does), while the solid curve includes *all* sources of ≥ 3 jet events, including especially $\widetilde{W}_1^+\widetilde{W}_1^-$ and $\widetilde{W}_1^{\pm}\widetilde{Z}_2$ production. The points and error bars correspond to our actual signal rate computation, along with statistical error. We also show the 95% CL limits as computed by adding maximally allowed signal cross section with SM error either linearly (BG2) or in quadrature (BG1). We see that, depending on how the signal and background levels are interpreted, squark masses in the 100 GeV range may or may not be allowed by Run 1 analyses. For the case shown, $m_{\tilde{u}_R} \sim 100 - 120\,\mathrm{GeV}$ would be excluded. If instead we take a case with larger $m_{1/2} = 99.5\,\mathrm{GeV}$, $m_{16}(1) = 1297.1\,\mathrm{GeV}$, $m_{16}(3) = 8767.2\,\mathrm{GeV}$, $m_{10}/m_{16}(3) = 1.238$, $M_D/m_{16}(3) = 0.356$, $\tan\beta = 50.79$ and $A_0/m_{16}(3) = -2.13$, then we generate a model with $R = 1.02$, $\Omega_{\widetilde{Z}_1}h^2 = 0.13$, $m_{\tilde{u}_R} = 117\,\mathrm{GeV}$ but with heavier charginos and neutralinos, with $\sigma(3-\mathrm{jets}+\not{E}_T) = 203\,\mathrm{fb}$, which is below both BG1 and BG2 levels. In figure 7, we show model points in the $m_{\widetilde{Z}_1}$ *vs.* $m_{\tilde{u}_R}$ plane. In this case, we see the mass gap $m_{\tilde{u}_R} - m_{\widetilde{Z}_1}$ ranging from 25-50 GeV. (The bands are due to sampling of $m_{1/2}$ in steps of 5 GeV, while other parameters are varied continuously.) Solutions with a large mass gap are more likely to give large observable rates at Tevatron searches, while solutions with small mass gaps will be more difficult to see, since there is less visible energy released in the squark decays.

An overall assessment seems to be that the two light squark scenario from Yukawa unified models with generational non-universality is at the edge of exclusion from previous

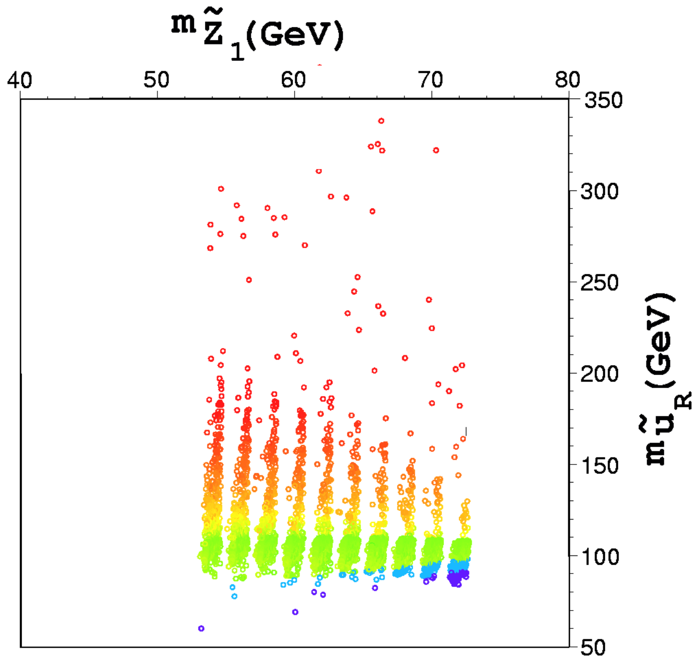

Figure 7: Scan of SO(10) *HS* models with generational non-universality, in the $m_{\tilde{u}_R}$ *vs.* $m_{\widetilde{Z}_1}$ plane. The colors of the dots correspond to the χ^2 value as listed in figure 4.

Tevatron searches using Run 1 data. However, given that Run 2 has already amassed $\sim 400\ \mathrm{pb}^{-1}$ of data, a new dedicated analysis by CDF and/or D0 of ≥ 2−jets + $\not{E}_T$ events arising from squark pair production will likely either rule in or rule out this scenario with two light squarks with mass in the 90-120 GeV range.

We mention here that there also exist Tevatron searches for $p\bar{p} \to \tilde{t}_1\bar{\tilde{t}}_1X$ production [43], where it is assumed that $\tilde{t}_1 \to c\widetilde{Z}_1$ 100% of the time. These analyses by CDF [44] and D0 [45] require a tagged charm jet, so would not apply to $\tilde{u}_R\bar{\tilde{u}}_R$ production. They do apply to $\tilde{c}_R\bar{\tilde{c}}_R$ production. However, the best limit (by CDF) requires $m_{\widetilde{Z}_1} < 50\,\mathrm{GeV}$, which is too light to apply to the scenario given here: see figure 7. However, a new search for $c\bar{c}+\not{E}_T$ events by CDF and D0 using Run 2 data should be able to probe the parameter space of the scenario listed in this paper.

Another consequence of light squarks is that they can mediate neutralino-nucleon scattering at observable rates via squark exchange in s and u-channel quark-neutralino subprocess diagrams. In figure 8, we plot various SO(10) *HS* models with Yukawa unification to better than 20%, in the $\log_{10}\sigma(\widetilde{Z}_1 p)$ *vs.* $\Omega_{\widetilde{Z}_1}h^2$ plane. We use the neutralino-nucleon scattering rate code developed in ref. [46]. The colors of the dots correspond to the χ^2 value as presented in figure 4. We see that the models with low relic density falling within the WMAP acceptable range also have rather large values of neutralino-nucleus scattering cross section. In fact, the purple (blue) dots are already excluded by Edelweiss [47] (CDMS II [48]) searches. CDMS II is expected to ultimately probe neutralino-proton scalar cross sections as low as 10^{-8} pb. This essentially covers the complete range of models with acceptable $\Omega_{\widetilde{Z}_1}h^2$ as obtained from Yukawa unified models with light $\tilde{u}_R$ and $\tilde{c}_R$ squarks.

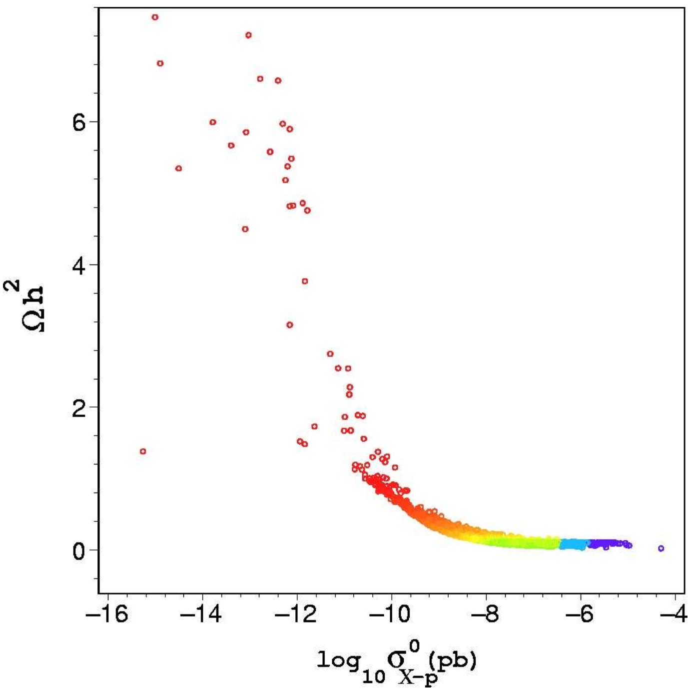

Figure 8: Scan of SO(10) HS models with generational non-universality, in the $\sigma(\widetilde{Z}_1 p)$ vs. $\Omega_{\widetilde{Z}_1}h^2$ plane. The colors of the dots correspond to the χ^2 value computed from $\Omega_{\widetilde{Z}_1}h^2$, $BF(b \to s\gamma)$ and a_μ^{SUSY}.

3. Gaugino mass non-universality

Aside from generational non-universality in SO(10) models, it is also possible to have non-universality of gaugino masses. In a general non-renormalizable SUSY gauge theory, the gaugino masses arise from the term

$$\mathcal{L} \ni -\frac{1}{4}\int d^2\theta_L f_{AB}(\hat{S})\overline{\hat{W}_A^c}\hat{W}_B\,, \tag{3.1}$$

where we have introduced the gauge kinetic function (GKF) $f_{AB}(\hat{S})$ which is an analytic function of chiral superfields $\hat{S}_i$, and where $\hat{W}_A$ are gauge superfields. In supergravity theories, the GKF can be expanded as

$$f_{AB} = \delta_{AB} + \frac{\hat{\phi}_{AB}}{M_P} + \cdots\,, \tag{3.2}$$

where, to maintain gauge invariance, $\hat{\phi}_{AB}$ transforms as the symmetric product of two adjoints. If the auxiliary field $F_{\phi AB}$ of $\hat{\phi}_{AB}$ obtains a SUSY breaking (and possibly gauge symmetry breaking) vev, then gaugino mass terms are induced:

$$\mathcal{L} \ni \frac{\langle F_{\phi AB}\rangle}{M_P}\overline{\lambda}_A\lambda_B + \cdots\,. \tag{3.3}$$

In the case of SU(5) GUT theories [50], the product of two adjoints can be decomposed as

$$(\mathbf{24}\times\mathbf{24})_{\rm sym} = \mathbf{1}+\mathbf{24}+\mathbf{75}+\mathbf{200}\,, \tag{3.4}$$

where only the singlet represention leads to universal gaugino masses. The higher representations lead to relations amongst the GUT scale gaugino masses which are dictated by group theory, and are listed in ref. [50]. In the general case, the field $\hat{\phi}_{AB}$ could transform as a linear combination of the above representations, in which case the three MSSM gaugino masses at $Q = M_{\rm GUT}$ would be essentially free parameters. In the case of SO(10) GUTS [51, 52], we have

$$(\mathbf{45} \times \mathbf{45})_{\rm sym} = \mathbf{1} + \mathbf{54} + \mathbf{210} + \mathbf{770}\,. \tag{3.5}$$

For our purposes here, we will not adopt any specific GUT symmetry breaking pattern or set of gaugino masses, but instead merely enlarge the parameter space by allowing each MSSM gaugino mass as a free parameter. Given this freedom, it is then possible to take any of the Yukawa unified solutions generated by Auto *et al.* [22] and vary the gaugino masses to see if an acceptable relic density solution is found. The only problem of concern is whether doing so will destroy the Yukawa coupling unification, or lead to models which are violated by constraints such as those imposed by LEP2 searches for sparticles. We note here that an alternative approach is to allow restricted, or even complete freedom of gaugino masses, and to explore whether Yukawa coupling unification is possible. That approach has been followed in the context of 5-d gaugino-mediated SUSY breaking models [53], where all scalar masses are set to zero at $M_c \sim M_{\rm GUT}$; we refer to the work of Balazs and Dermisek for more details [54].

We have examined the positive μ Yukawa unified solutions presented in table 1 of [22] with regard to variation of the GUT scale gaugino masses M_1 and M_2. In the case of decreasing M_2, ultimately the lightest neutralino makes a transition from being bino-like to being wino-like. Wino-like LSPs have a large annihilation and co-annihilation rate into SM particles, and give a relic density almost always much lower than the WMAP bound [55]. However, in lowering M_2 to the point of achieving a valid $\Omega_{\widetilde{Z}_1}h^2$, we find in these five cases that the chargino mass always falls below the LEP2 limits, so that the model becomes excluded by LEP2 rather than WMAP. The other case, that of varying the $U(1)_Y$ gaugino mass M_1, leads to models that can satisfy the WMAP bound while being consistent with other constraints including those from LEP2.

In figure 9, we show the relic density for four of the points from table 1 of [22] versus variation in GUT scale gaugino mass M_1. In all cases there is a similar structure: for generic values of M_1, the relic density lies beyond the WMAP limits. However, there exists a double dip structure where the relic density may or may not fall below the WMAP limit, and then at high M_1, the relic density drops to very low values which would imply non-neutralino cold dark matter. The high M_1 behavior occurs because as M_1 is increased, ultimately $M_1 > M_2$, and the LSP becomes wino-like, although in this case, since M_2 is fixed, the chargino mass stays fixed beyond the LEP2 mass limits. At lower M_1 values, the double dip structure occurs when the value of $2m_{\widetilde{Z}_1}$ drops below the h or the Z resonance. In these cases, the neutralinos can efficiently annihilate at high rates through these resonances in the early universe, leading to lower relic density values. In some of the cases, the diminution is sufficient to drop below WMAP limits, while in others (those with very heavy scalars, and hence with a large μ parameter), the diminution is insufficient to save the model. In

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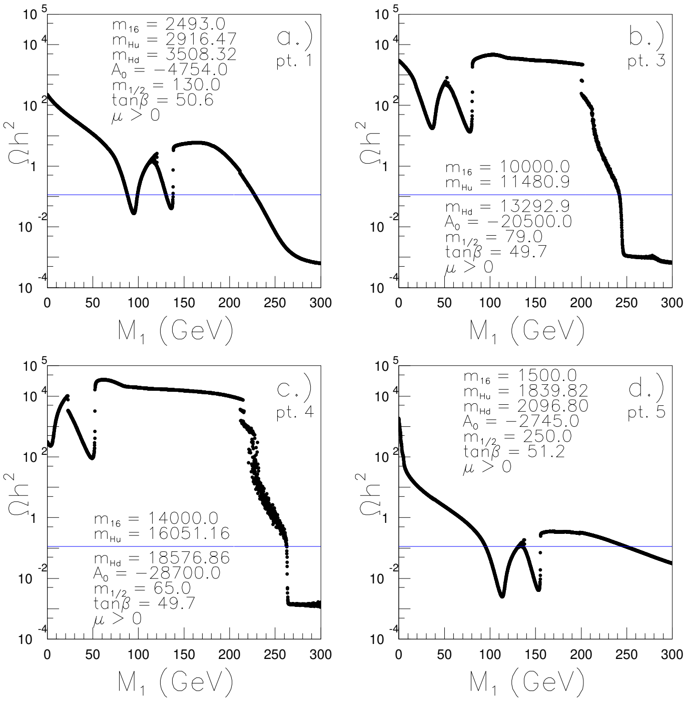

Figure 9: Plot of $\Omega_{\widetilde{Z}_1} h^2$ versus GUT scale gaugino mass M_1 for four case studies extracted from table 1 of ref. [22]. We also show as a horizontal line the relic density limit demanded by WMAP analysis.

each case, we have found that the Yukawa coupling unification changes by less than a per cent for variation of the M_1 parameter over the range indicated in figure 9.

4. Conclusions

Supersymmetric GUTs based on the SO(10) gauge group are highy motivated theories for physics beyond the standard model. Many of these models also predict unification of the $t - b - \tau$ Yukawa couplings as well. The restriction of third generation Yukawa coupling unification places stringent constraints on the parameter space of supersymmetric models, and leads to some new and interesting predictions for new physics. In this paper, we restrict ourselves to models with a positive superpotential μ parameter, which is favored by $BF(b \to s\gamma)$ and $(g-2)_\mu$ measurements. In these models, Yukawa unified sparticle mass solutions with $R < 1.05$ can be found, but only in the HS case, and for very large $m_{16} \sim 5 - 15\,\text{TeV}$ case. This also leads generally to TeV scale values of m_A and μ, and these numbers, along with TeV scale masses for matter scalars, act to suppress neutralino annihilation in the early universe, and hence to predictions of a value of $\Omega_{\widetilde{Z}_1} h^2$ far in excess of WMAP allowed values.

In this paper, we have explored two methods in which to reconcile Yukawa unified sparticle mass solutions with the neutralino relic density. The first case is to allow splitting of the third generation of scalars from the first two generations. By decreasing the first and second generation scalar masses, we obtain sparticle mass solutions with very light $\tilde{u}_R$

and $\tilde{c}_R$ squark masses, in the 90-120 GeV range. The origin of the light squarks comes from the S term in the one-loop RGEs, which is non-zero and large in the case of multi-TeV valued split Higgs masses. A search for just two light squarks at the Fermilab Tevatron collider using the new Run 2 data should be able to either verify or disprove this scenario. In addition, the light squarks give rise to large rates for direct detection of dark matter, and should give observable rates for searches at CDMS II.

Some possible criticisms of the model presented in section 2 is that 1. a degree of fine-tuning is required in order to obtain the correct relic density, and 2. the large trilinear soft breaking A parameters may lead to charge and/or color breaking minima in the scalar potential. We would concur with the first of these criticisms. However, our goal in this paper was to examine whether or not a relic density in accord with WMAP could be generated in Yukawa unified models with $\mu > 0$, irregardless of fine-tuning arguments. Regarding point 2, we note that beginning with Frere *et al.*[56], bounds have been derived on soft breaking parameters of the form $|A_u|^2 \leq 3(m_Q^2 + m_U^2 + m_{H_u}^2)$ by trying to avoid charge and/or color breaking minima of the scalar potential. A complete analysis must go beyond the tree level and include at least 1-loop radiative corrections to the scalar potential [57]. We do not include such an analysis here in part because of the philosophical question of whether false vacua are allowed, and also because a simple solution of the problem may arise by taking the first two generation A_0 parameters to be light, and non-degenerate with the third generation A parameter.

The other scenario we examined was the case of non-degenerate gaugino masses at the GUT scale. Beginning with any of the Yukawa unified solutions found in an earlier study, we find that by dialing M_1 to large enough values, a (partially) wino-like LSP can be generated, with a relic density in accord with WMAP allowed values. Also, we showed that M_1 can be dialed so that the value of $2m_{\widetilde{Z}_1}$ sits near the Z or h resonance, thus enhancing annihilation rates of the $\widetilde{Z}_1$ in the early universe. Depending on the model considered, this may or may not be enough to bring the neutralino relic density into accord with the WMAP measured value.

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